Correlation functions in 1D Bose gases : density fluctuations and momentum distribution

Isabelle Bouchoule, Julien Armijo, Thibaut Jacqmin, Tarik Berrada, Bess Fang, Karen Kheruntsyan[2] and T. Roscilde[3]

Institut d'Optique, Palaiseau.
[2] Brisbane, Australia
[3] ENS Lyon

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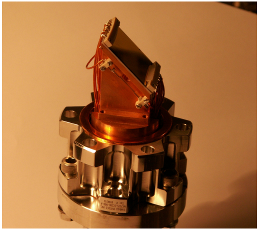
Correlations in 1D Bose gases

- Correlation functions : important characterisation of quantum gases
- Physics in reduced dimension
  - very different from 3D (absence of BEC in 1D ideal Bose gases)
  - Enhanced fluctuations : no TRLO
  - Physics governed by interactions
- Cold atom experiments : powerful simulators of quantum gases. Reduced dimension achieved by strong transverse confinement. Atom chip experiment : 1D configuration naturally realised.



Outline

Outline

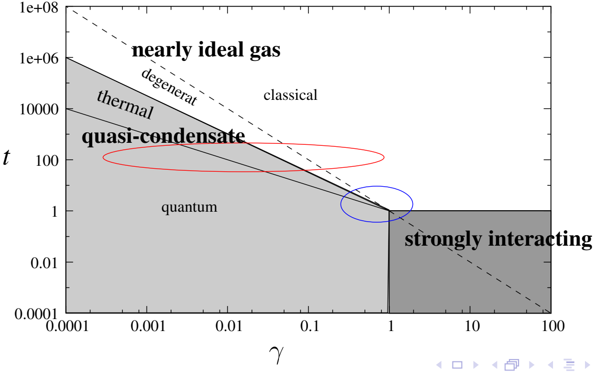
1D Bose gas with repulsive contact interaction

$$H = -\frac{\hbar^2}{2m}\int dz\psi^+\frac{\partial^2}{\partial_z^2}\psi + \frac{g}{2}\int dz\psi^+\psi^+\psi\psi,$$

Exact solution : Lieb-Liniger   Thermodynamic : Yang-Yang (60') n, T

Length scale : $l_g = \hbar^2/mg$, Energy scale $E_g = g^2m/2\hbar^2$

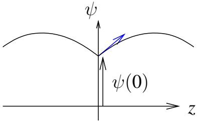
Parameters : $t = T/E_g$, $\gamma = 1/nl_g = mg/\hbar^2 n$



Strongly interacting 1D Bose gas

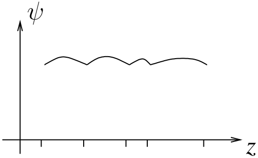
- 2 body scattering wave function $\psi(z) = \cos(kz + \varphi), E = \hbar^2 k^2/m$

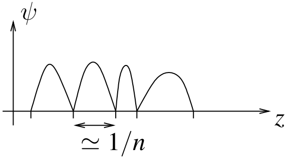
If $E \gg E_g, |\psi(0)| \simeq 1$
If $E \ll E_g, |\psi(0)| \ll 1$



$$E_g = mg^2/2\hbar^2$$

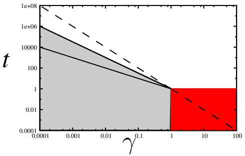
- Many body system





$g^{(2)}(0) \simeq 0$

Fermionization



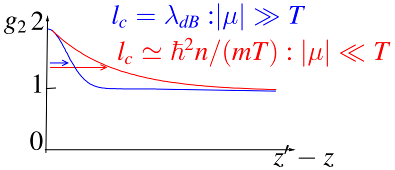
$E = gn \gg E_g$
$\Rightarrow \gamma \ll 1$

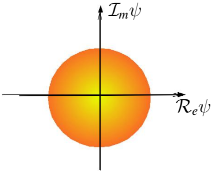
$E = \hbar^2 n^2/m \ll E_g$
$\Rightarrow \gamma \gg 1$

Strongly interacting regime : $\gamma \gg 1, t \ll 1$

Nearly ideal gas regime : bunching phenomena

- Two-body correlation function $g_2(z) = \langle \psi_z^+ \psi_0^+ \psi_0 \psi_z \rangle / n^2$





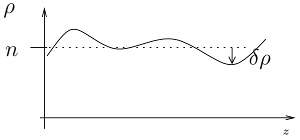
- Bunching effect $\rightarrow$ density fluctuations.
  $\langle \delta n(z)\, \delta n(z') \rangle = \langle n \rangle^2 (g_2(z' - z) - 1) + \langle n \rangle \delta(z - z')$

- Bunching : correlation between particles. Quantum statistic

- Field theory $\psi = \sum \psi_k e^{ikz}$, $n = |\psi|^2$: speckle phenomena

Transition towards quasi-condensate

- Repulsive Interactions → Density fluctuations require energy



$H_{int} = \frac{g}{2} \int dz \rho^2 \Rightarrow \delta H_{int} > 0$

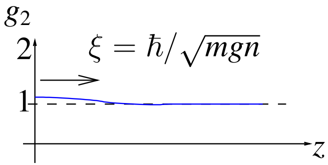
Reduction of density fluctuations at low temperature/high density

weakly interacting : $\gamma = mg/\hbar^2 n \ll 1$

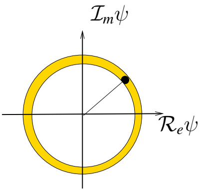
Cross-over : $\frac{1}{N} H_{int} \propto gn \simeq |\mu|$

$\mu = mT^2/2\hbar^2 n^2, \Rightarrow$ $\boxed{T_{c.o.} \simeq \frac{\hbar^2 n^2}{2m}\sqrt{\gamma}}$, $\boxed{n_{c.o.} \propto T^{2/3}}$ Transition for a degenerate gas

- For $T \ll T_{c.o.}$: quasi-bec regime, $g^{(2)} \simeq 1$



$T > gn$



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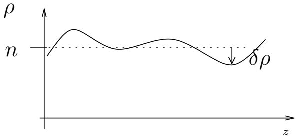

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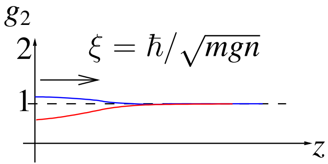
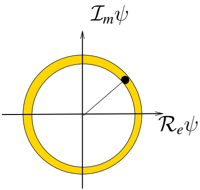
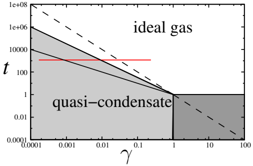


$T > gn$

$T < gn$

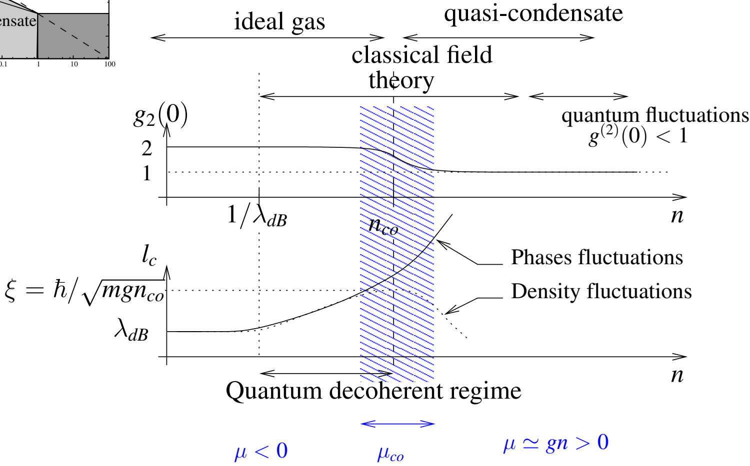


1D weakly interacting homogeneous Bose gas



$$t = 2\hbar^2 T/(mg^2)$$

$$n_{co} = \frac{1}{\lambda_{dB}} t^{1/6}$$



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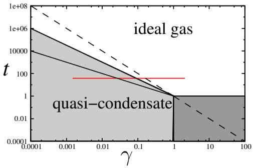
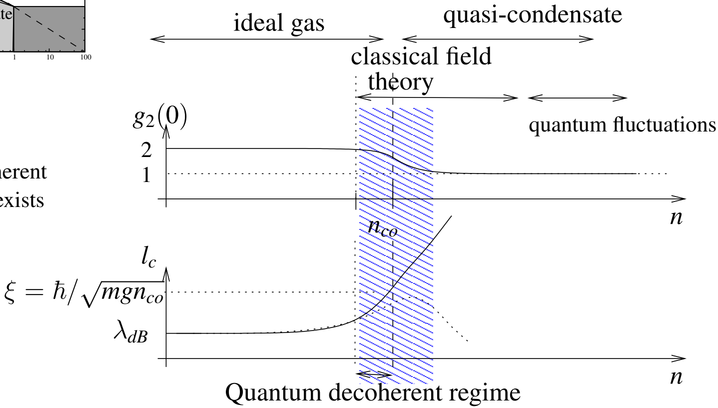


$$t = 2\hbar^2 T/(mg^2)$$

$$n_{co} = \frac{1}{\lambda_{dB}} t^{1/6}$$

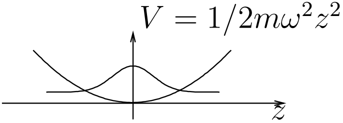
large transition quantum decoherent regime barely exists for $t < 1000$



$\mu < 0$ $\mu \simeq gn > 0$

Cross-over towards quasi-bec in a one-dimensionnal gas trapped in a harmonic potential

Local density approach :
$\mu(z) = \mu_0 - m\omega^2 z^2/2$

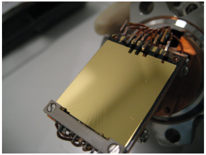


Local correlations properties :
that of a homogeneous gas with $\mu = \mu(z)$.
Validity : $l_c \ll \frac{1}{n}\frac{dn}{dz}$.
At quasi-condensation transition : $l_c = \xi$
$\Rightarrow T \gg \omega_\perp \left(\frac{\omega}{\omega_\perp}\right)^{3/2} \frac{l_\perp}{a}$
Easilly fulfilled experimentally.

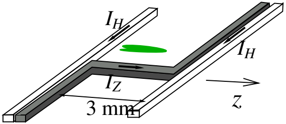
Outline

Realisation of very anisotropic traps on an atom chip

Magnetic confinement of $^{87}R_b$ by micro-wires



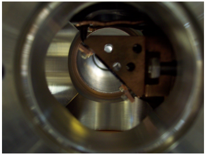
H-shape trap

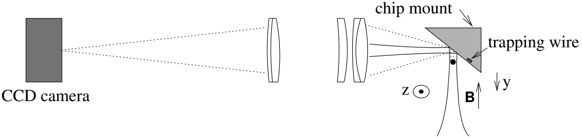


$$\begin{cases} \omega_\perp/2\pi = 3 - 4 \text{ kHz} \\ \omega_z/2\pi = 5 - 10 \text{ Hz} \end{cases}$$

$1D : T, \mu \ll \hbar\omega_\perp$

$g = 2\hbar\omega_\perp a$

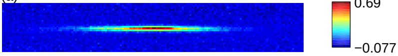




In-situ images

absolute calibration

(a)



$T \simeq 400 - 15nK$

$\simeq 3.0 - 0.1\hbar\omega_\perp$

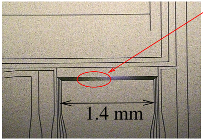
$N \simeq 5000 - 1000.$

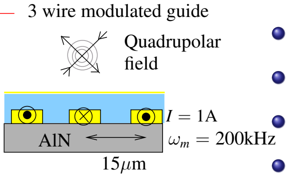
$t \simeq 80 - 1000$

Weakly interacting gases

Reaching strongly interacting gases on an atom-chip

- A new atom-guide





- Roughness free (modulation)
- Strong confinement : 1 – 150 kHz
- Diverse longitudinal potentials
- Dynamical change of g

- Approaching strong interactions

$\omega_\perp/(2\pi) = 19$ kHz, $\omega_z = 7.5$ Hz

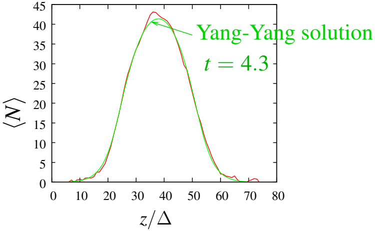
Experimental sequence :

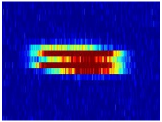
⋆ Cooling at moderate $\omega_\perp$

⋆ Increase $\omega_\perp$

Problem : exess of heating

(Entropy not preserved)





Outline

Density fluctuation measurements

- Statistical analysis over hundreds of images

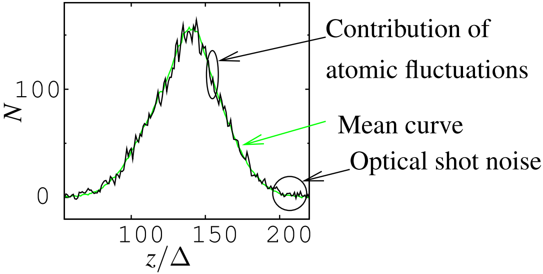
$l_c, \Delta \ll L$

$\rightarrow$ local density approximation

δN binned according to $\langle N \rangle$

$\langle \delta N^2 \rangle$ versus $n = \langle N \rangle / \Delta$

Optical shot noise substracted



$\langle \delta N^2 \rangle$: Two-body correlation function integral

$$\langle \delta N^2 \rangle = \langle N \rangle + n^2 \int \int dz (g_2(z - z') - 1)$$

$$l_c \ll \Delta \Rightarrow \langle \delta N^2 \rangle = \langle N \rangle + \langle N \rangle n \int dz (g_2(z - z') - 1)$$

$\Rightarrow$ Thermodynamic quantity

$$\boxed{\langle \delta N^2 \rangle = k_B T \Delta \frac{\partial n}{\partial \mu}}$$

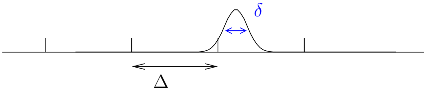
Effect of finite spatial resolution

- Absorption of an atom spreads on several pixels

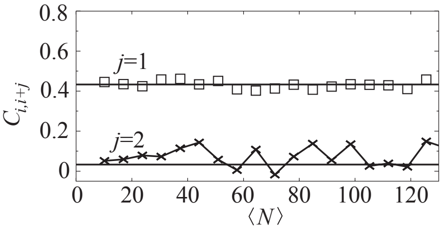
$\rightarrow$ blurring of the image

Decrease of fluctuations :

$\langle \delta N^2 \rangle = \kappa_2 \langle \delta N^2 \rangle_{\text{true}}$



$\rightarrow$ Correlation between pixels

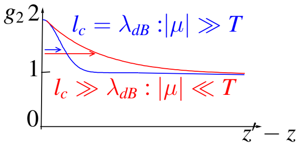


δ deduced from measured correlations between adjacent pixels

κ_2 deduced

Expected behavior in asymptotic regimes

- Ideal gas regime



$$\langle \delta N^2 \rangle = \langle N \rangle + \langle N \rangle n \underbrace{\int dz (g_2(z) - 1)}_{l_c}$$

▷ Non degenerate gas : $nl_c \ll 1$

$$\langle \delta N^2 \rangle \simeq \langle N \rangle$$

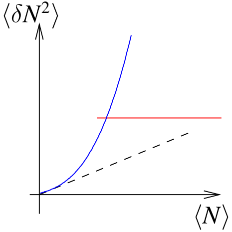
▷ Degenerate gas : $nl_c \gg 1$

$$\langle \delta N^2 \rangle \simeq \langle N \rangle n l_c = \langle N \rangle^2 l_c / \Delta$$

- Quasi-bec regime

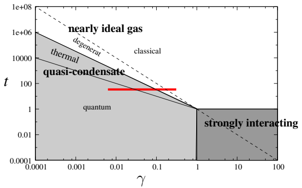
Thermodynamic : $\mu \simeq gn$

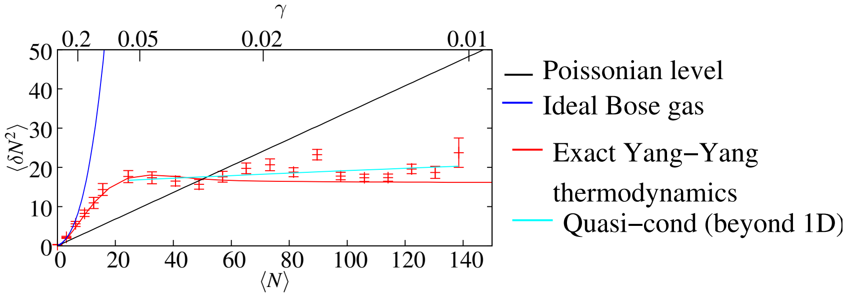
$\Rightarrow \langle \delta N^2 \rangle = \Delta T / g$



Experimental results

$T = 15$ nK$\simeq \hbar\omega_\perp/10$
$t \simeq 64$
$\mu \simeq 30$ nK$\simeq 0.2\ \hbar\omega_\perp$





- Strong bunching effect in the transition region
- Quasi-bec both in the thermal and quantum regime

Quasi-condensate in the quantum regime

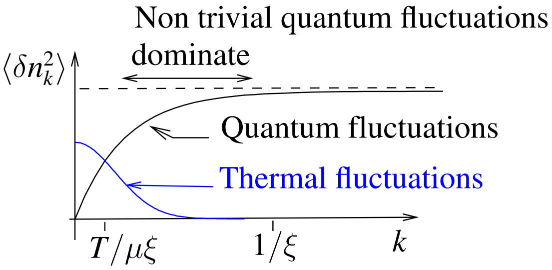
We measure : $\boxed{\langle \delta N^2 \rangle < \langle N \rangle}$

$\langle \delta N^2 \rangle = \langle N \rangle + \langle N \rangle n \int dz (g_2(z) - 1)$ $\Rightarrow g^{(2)} < 1$: Quantum regime

- However we still measure thermal excitations

Shot noise term, removed from the $g^{(2)}$ fonction, IS quantum.

Low momentum phonons ($k \ll T/\mu\xi$) : high occupation number

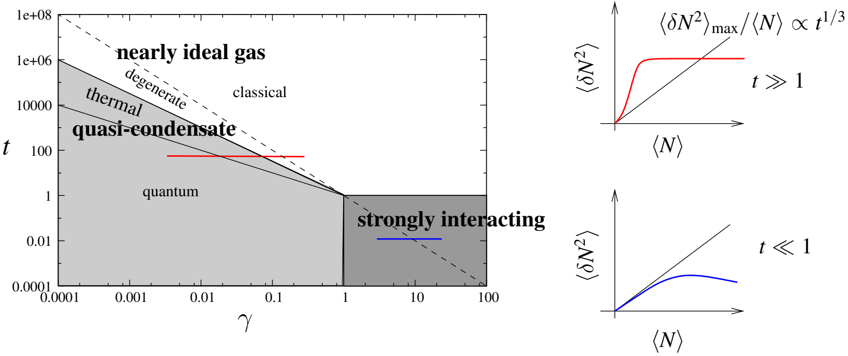


For our datas :

$l_T = (T/\mu\xi)^{-1} < 450$ nm

$l_T \ll \Delta, \delta$

From weakly to strongly interacting gases

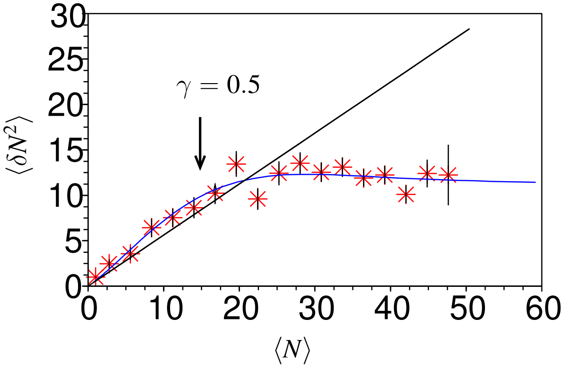


- smaller t : smaller bunching $\langle \delta N^2 \rangle_{max} / \langle N \rangle \propto t^{1/3}$
- $t \ll 1$: Fermi behavior $\rightarrow$ from poissonian to sub-poissonian. No bunching anymore

t small, γ large $\Rightarrow$ g large $\Rightarrow$ large $\omega_\perp$

Density fluctuations close to the strongly interacting regime

Moderate compression : $\omega_\perp/2\pi = 18.8$ kHz
$T = 40$ nK$\simeq \hbar\omega_\perp/20$ $t \simeq 5$
$\mu/T \simeq 1.9$



No bunching seen anymore, at a level of 20%.
Behavoir close to that of a Fermi gas

Outline

1D-3D Crossover in the quasi-bec regime

$\mu \ll \hbar\omega_\perp$ $(na \ll 1) \rightarrow$ Pure 1D quasi-bec

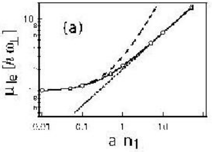
$\mu \gtrsim \hbar\omega_\perp$: 1D$\rightarrow$ 3D behavior

Transverse breathing associated with a longitudinal phonon has to be taken into account.

Thermodynamic argument : $\mathrm{Var}(N) = k_B T \left.\frac{\partial N}{\partial \mu}\right)_T$

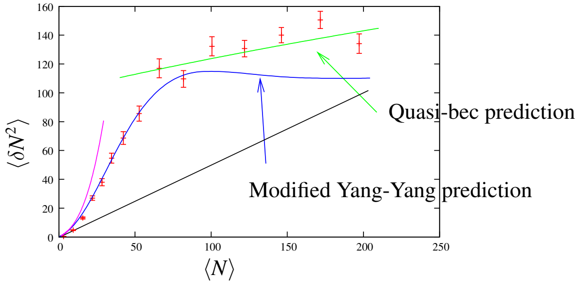
Heuristic equation :

$$\mu = \hbar\omega_\perp\sqrt{1 + 4na}$$



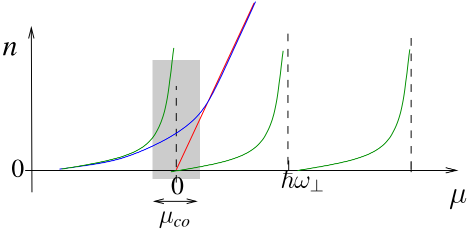
Efficient thermometry

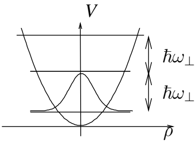
$T = 96$ nK $\simeq 0.5\hbar\omega_\perp$



Modified Yang-Yang model

If $\mu_{co} \ll \hbar_{\omega_\perp}$, transversally excited states behave as ideal Bose gases.





$$\mu_{co} \simeq E_g^{1/3} T^{2/3} = \left(mg^2/\hbar^2\right)^{1/3} T^{2/3} \Rightarrow \mu_{co}/(\hbar\omega_\perp) \simeq \left(\frac{T}{\hbar\omega_\perp}\frac{a}{l_\perp}\right)^{2/3}$$

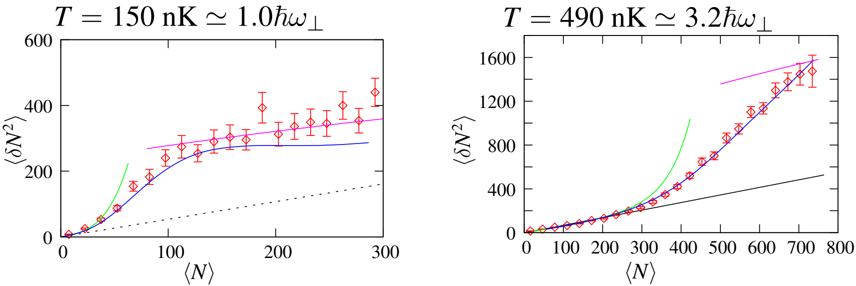
For our parameters : $a/l_\perp \simeq 0.025$

Modified Yang-Yang model :

- Transverse ground state : Yang-Yang thermodynamic
- Excited transverse states : ideal 1D Bose gases

First introduced by Van Druten and co-workers : Phys. Rev. Lett. 100, 090402 (2008)

Modified Yang-Yang model



Very good agreement

Outline

Third moment of density fluctuations

Non gaussian fluctuations

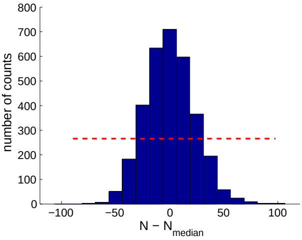
We look at $\boxed{\langle \delta N^3 \rangle}$

- Motivation

Learn about 3 body correlations

- A thermodynamic quantity :

$$\langle \delta N^3 \rangle = \Delta T^2 \frac{\partial^2 n}{\partial \mu^2}$$

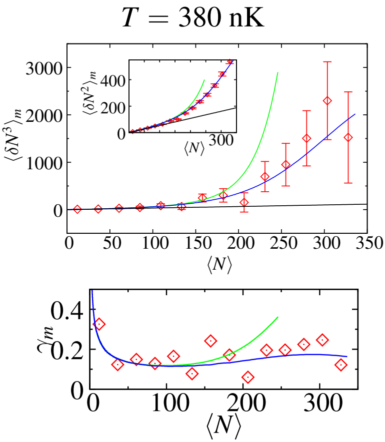


- Effect of finite spatial resolution :

$\langle \delta N^3 \rangle = \kappa_3 \langle \delta N^3 \rangle_{\rm true}$, κ_3 depends on resolution.

κ_3 infered from neighbour pixels correlation.

Third moment of density fluctuations



Measured 3rd moment compatible with MYY.

Skweness vanishes in quasi-bec regime (as expected)

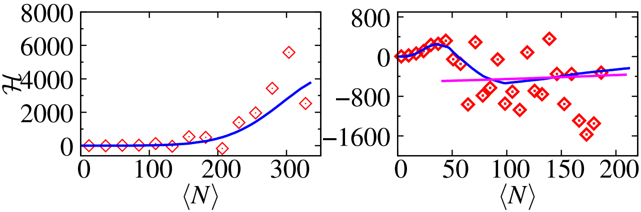
$\gamma_m = \langle \delta N^3 \rangle / \langle \delta N^2 \rangle^{3/2}$

Three-body correlations

$\langle \delta N^3 \rangle$ function of $g^{(2)}$ and $g^{(3)}$. $g^{(3)}$ function contains $g^{(2)}$

We measure :

$$\begin{cases} \mathcal{H} = \langle \delta N^3 \rangle + 2\langle N \rangle - 3\langle \delta N^2 \rangle = n^3 \int\int\int h(z_1,z_2,z_3), \\ h(z_1,z_2,z_3) = g^{(3)}(z_1,z_2,z_3) - g^{(2)}(z_1,z_2) - g^{(2)}(z_1,z_3) - g^{(2)}(z_2,z_3) + 2 \end{cases}$$



Outline

Momentum distribution measurements : motivation

Momentum distribution : TF of first order correlation function

$$n_k = n \int du g^{(1)}(u) e^{iku}$$

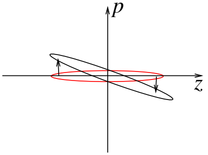
Cannot be derived from the equation of state.
Its measurement opens perspectives (study of dynamics for example)

Problem : global measurement $\rightarrow$ averaged over many different situations in presence of a smooth longitudinal potential

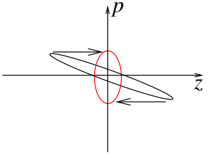
Focussing techniques

Momentum distribution measurement

- Short kick of a strong harmonic potential
$\Rightarrow \delta p = -Az$



- Free fligh until focuss



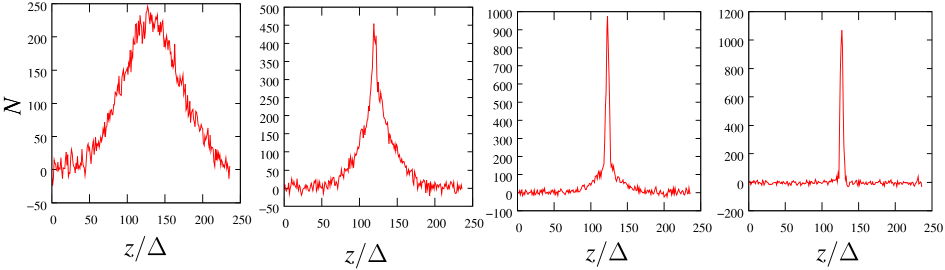
Final spatial distribution : initial momentum distribution, averaged over the initial position

Experiment well adapted :

- longitudinal confinement undependent on transverse one
- Purely harmonic potential (up to order z^5)

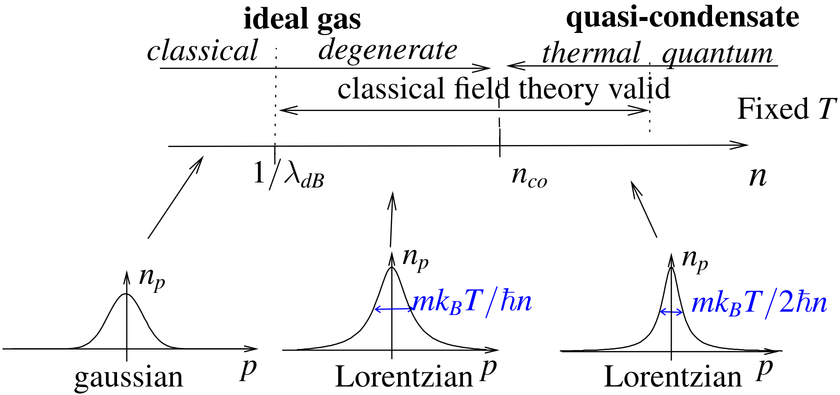
Focussing

Images taken at focus ($t_v = 15$ms) for different initial temperatures (initial RF knife position)



Expected results in limiting cases

Momentum distribution not knowned exactly.



Beyond this approach, some knowed results :

⋆ $1/p^4$ tail

⋆ Mean kinetic energy (second moment of n_p) via Yang-Yang

1D classical field calculation

$\psi(z)$: complex field of energy functionnal

$$E[\{\Psi\}] = \int dz \left[\frac{\hbar^2}{2m} \left| \frac{d\Psi}{dz} \right|^2 + \frac{g}{2} |\Psi|^4 - \mu |\Psi|^2 \right]$$

No high energy divergence (as in higher dimension).
Mapped to a Schrödinger equation, evolving in imaginary time

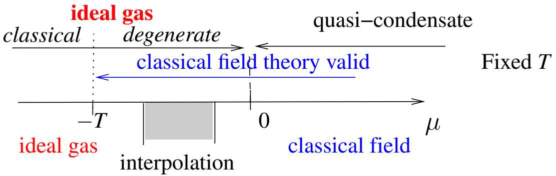
$$\hat{H} = \frac{\hat{p}^2}{2M} + \frac{\hbar}{k_B T} \left[\frac{g}{2} (\hat{x}^2 + \hat{y}^2)^2 - \mu (\hat{x}^2 + \hat{y}^2) \right], \qquad M = \frac{\hbar^3}{m k_B T}$$

For $L \to \infty$, only ground state contribute to $g^{(1)}$:

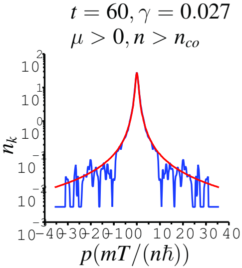
$$g^{(1)}(z) = \langle \phi_0 | (\hat{x} - i\hat{y}) e^{-\hat{H}z/\hbar} (\hat{x} + i\hat{y}) | \phi_0 \rangle$$

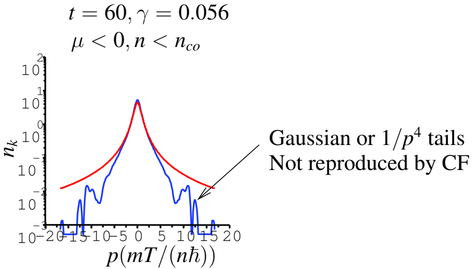
Very fast calculation (as opposed to Monte Carlo sampling).

Comparison with exact calculations

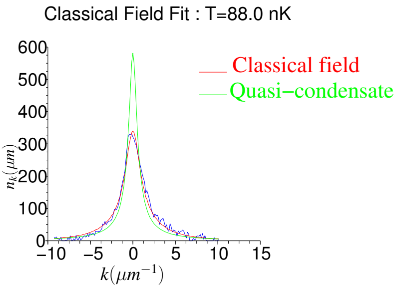


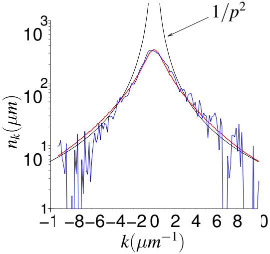
Comparison with quantum Monte Carlo calculation

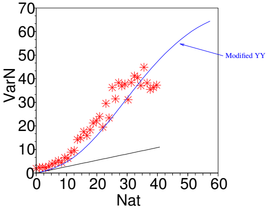




Result : degenerate ideal Bose gas regime

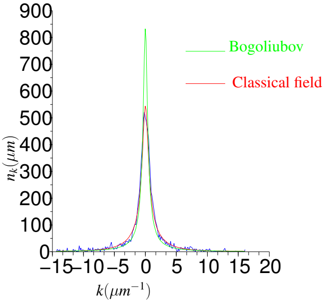


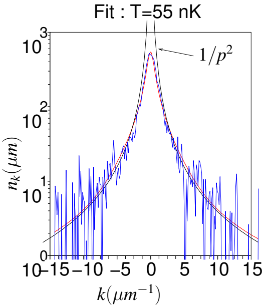


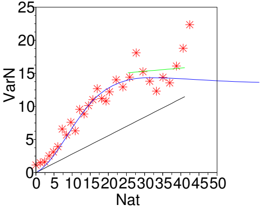


- In agreement with insitu density fluctuations
- Good agreement with CF
- Lorentzian behavior : $1/p^2$ tail

Result : quasi-condensate regime







- In agreement with insitu density fluctuations
- Good agreement with CF
- Lorentzian behavior : $1/p^2$ tail
- not in agreement with Bogoliubov

Conclusion and prospetcs

Conclusion

- Precise density fluctuation measurement
  - Good thermometry
  - Investigation of the quasi-condensation transition
  - Strong anti-bunching
  - Higher order correlation functions
  - Dimensional crossover
- Momentum distribution measurement

Prospects

- Investigation of out-of-equilibrium situations : dynamic following a quench, relaxation towards non thermal states
- Using tomography to gain in spatial resolution and investigate $g^{(2)}(r)$
- Investigating the physics of the Mott transition in 1D using the probes we developped : pinning transition.