Correlation functions in 1D Bose gases: density fluctuations and momentum distribution

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Correlations in 1D Bose gases

- **Correlation functions**: important characterisation of quantum gases
- **Physics in reduced dimension**
  - very different from 3D (absence of BEC in 1D ideal Bose gases)
  - Enhanced fluctuations: no TRLO
  - Physics governed by interactions
- **Cold atom experiments**: powerful simulators of quantum gases. Reduced dimension achieved by strong transverse confinement. Atom chip experiment: 1D configuration naturally realised.
Outline

1. Regimes of 1D Bose gases
2. Experimental apparatus
3. Density fluctuations
   - Measurements
   - Weakly interacting regime: quasi-condensation and sub-poissonian fluctuations
   - Entering the strongly interacting regime
4. Beyond 1D physics
   - Quasi-bec
   - Contribution of excited transverse states in the crossover
5. Third moment of density fluctuations
6. Momentum distributions of a 1D gas
   - Focussing technics
   - Classical field analysis
   - Results
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1D Bose gas with repulsive contact interaction

\[ H = -\frac{\hbar^2}{2m} \int dz \psi^+ \frac{\partial^2}{\partial z^2} \psi + \frac{g}{2} \int dz \psi^+ \psi^+ \psi \psi, \]

Exact solution : Lieb-Liniger  
Thermodynamic : Yang-Yang (60’) \( n, T \)

Length scale : \( l_g = \hbar^2 / mg \), Energy scale \( E_g = \frac{g^2 m}{2\hbar^2} \)

Parameters : \( t = T / E_g, \gamma = 1 / nl_g = mg / \hbar^2 n \)
Strongly interacting 1D Bose gas

- 2 body scattering wave function \( \psi(z) = \cos(kz + \varphi), E = \hbar^2 k^2 / m \)

If \( E \gg E_g, |\psi(0)| \approx 1 \)
If \( E \ll E_g, |\psi(0)| \ll 1 \)

\( E_g = mg^2 / 2\hbar^2 \)

- Many body system

\( E = gn \gg E_g \)
\( \Rightarrow \gamma \ll 1 \)

\( E = \hbar^2 n^2 / m \ll E_g \)
\( \Rightarrow \gamma \gg 1 \)

Strongly interacting regime : \( \gamma \gg 1, t \ll 1 \)
Nearly ideal gas regime: bunching phenomena

- Two-body correlation function \( g_2(z) = \frac{\langle \psi_z^+ \psi_0^+ \psi_0 \psi_z \rangle}{n^2} \)

- Bunching effect \( \rightarrow \) density fluctuations.
  \[ \langle \delta n(z) \delta n(z') \rangle = \langle n \rangle^2 (g_2(z' - z) - 1) + \langle n \rangle \delta(z - z') \]

- Bunching: correlation between particles. Quantum statistic

- Field theory \( \psi = \sum \psi_k e^{ikz} \), \( n = |\psi|^2 \): speckle phenomena
Transition towards quasi-condensate

- Repulsive Interactions $\rightarrow$ Density fluctuations require energy

$$H_{\text{int}} = \frac{g}{2} \int dz \rho^2 \Rightarrow \delta H_{\text{int}} > 0$$

Reduction of density fluctuations at low temperature/high density
weakly interacting: $\gamma = mg/\hbar^2 n \ll 1$

Cross-over: $\frac{1}{N} H_{\text{int}} \propto gn \simeq |\mu|$

$$\mu = \frac{mT^2}{2\hbar^2 n^2}, \Rightarrow \ T_{\text{c.o.}} \simeq \frac{\hbar^2 n^2}{2m} \sqrt{\gamma}, \ n_{\text{c.o.}} \propto T^{2/3}$$

Transition for a degenerate gas

- For $T \ll T_{\text{c.o.}}$: quasi-bec regime, $g^{(2)} \simeq 1$

$$\xi = \hbar / \sqrt{mgn} \quad T > gn$$
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Transition for a degenerate gas

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$$g_2$$

$$\xi = \hbar / \sqrt{mgn}$$

$$T > gn$$

$$T < gn$$
1D weakly interacting homogeneous Bose gas

\[ t = \frac{2\hbar^2 T}{(mg^2)} \]

\[ n_{co} = \frac{1}{\lambda_{dB}} t^{1/6} \]

\[ \xi = \frac{\hbar}{\sqrt{mgn_{co}}} \]

Phases fluctuations
Density fluctuations
Quantum decoherent regime

\[ \mu < 0 \quad \mu_{co} \quad \mu \simeq gn > 0 \]
1D weakly interacting homogeneous Bose gas

Regimes of 1D Bose gases

Experimental apparatus

Density fluctuations

Beyond 1D physics

Third moment of density fluctuations

Momentum

large transition

quantum decoherent

regime barely exists

for $t < 1000$

ideal gas

quasi-condensate

classical field

theory

quantum fluctuations

$\xi = \hbar / \sqrt{mg n_{co}}$

$\lambda_{dB}$

$n_{co} = \frac{1}{\lambda_{dB}} t^{1/6}$

$g_2(0)$

2

1

$n_{co}$

Quantum decoherent regime

$\mu < 0$

$\mu \simeq gn > 0$
Cross-over towards quasi-bec in a one-dimensionnal gas trapped in a harmonic potential

Local density approach:
\[ \mu(z) = \mu_0 - m\omega^2 z^2 / 2 \]

Local correlations properties:
that of a homogeneous gas with \( \mu = \mu(z) \).

Validity: \( l_c \ll \frac{1}{n \frac{dn}{dz}} \).

At quasi-condensation transition: \( l_c = \xi \)

\[ \Rightarrow T \gg \omega_\perp \left( \frac{\omega}{\omega_\perp} \right)^{3/2} \frac{l_\perp}{a} \]

Easilly fulfilled experimentally.
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Realisation of very anisotropic traps on an atom chip

Magnetic confinement of $^{87}\text{Rb}$ by micro-wires

H-shape trap

$$\begin{aligned}
\left\{ \begin{array}{l}
\omega_{\perp}/2\pi = 3 - 4 \text{ kHz} \\
\omega_z/2\pi = 5 - 10 \text{ Hz}
\end{array} \right.
\end{aligned}$$

$1\text{D}: T, \mu \ll \hbar \omega_{\perp}$

$$g = 2\hbar \omega_{\perp} a$$

In-situ images

absolute calibration

(a)

$$\begin{aligned}
T &\approx 400 - 15nK \\
\approx 3.0 - 0.1\hbar \omega_{\perp} \\
N &\approx 5000 - 1000.
\end{aligned}$$

$t \approx 80 - 1000$

Weakly interacting gases
Reaching strongly interacting gases on an atom-chip

- A new atom-guide
  - 3 wire modulated guide
  - Quadrupolar field
  - AlN 15 µm
  - \( I = 1 \text{A} \)
  - \( \omega_m = 200\text{kHz} \)
  - Roughness free (modulation)
  - Strong confinement: 1 – 150 kHz
  - Diverse longitudinal potentials
  - Dynamical change of \( g \)

- Approaching strong interactions

Experimental sequence:
- Cooling at moderate \( \omega_\perp \)
- Increase \( \omega_\perp \)
Problem: excess of heating
(Entropy not preserved)

\[ \omega_\perp / (2\pi) = 19 \text{kHz}, \omega_z = 7.5 \text{Hz} \]

\( t = 4.3 \)

Yang-Yang solution
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Density fluctuation measurements

- Statistical analysis over hundreds of images

\[ l_c, \Delta \ll L \]

→ local density approximation

\[ \delta N \text{ binned according to } \langle N \rangle \]

\[ \langle \delta N^2 \rangle \text{ versus } n = \langle N \rangle / \Delta \]

Optical shot noise subtracted

\[ \langle \delta N^2 \rangle : \text{Two-body correlation function integral} \]

\[ \langle \delta N^2 \rangle = \langle N \rangle + n^2 \int \int dz (g_2(z - z') - 1) \]

\[ l_c \ll \Delta \Rightarrow \langle \delta N^2 \rangle = \langle N \rangle + \langle N \rangle n \int dz (g_2(z - z') - 1) \]

⇒ Thermodynamic quantity

\[ \langle \delta N^2 \rangle = k_B T \Delta \frac{\partial n}{\partial \mu} \]
Effect of finite spatial resolution

- Absorption of an atom spreads on several pixels
  → blurring of the image

  Decrease of fluctuations:
  \[
  \langle \delta N^2 \rangle = \kappa_2 \langle \delta N^2 \rangle_{\text{true}}
  \]

  → Correlation between pixels

  \[
  C_{i,i+j} = \begin{cases} 
  0.8 & j=1 \\
  0.6 & j=2 \\
  0.4 & \\
  0.2 & \\
  \text{0} & \end{cases}
  \]

  \[
  \langle N \rangle = \begin{cases} 
  0 & 0 \leq \langle N \rangle \leq 20 \\
  0.2 & 20 < \langle N \rangle \leq 40 \\
  0.4 & 40 < \langle N \rangle \leq 60 \\
  0.6 & 60 < \langle N \rangle \leq 80 \\
  0.8 & 80 < \langle N \rangle \leq 100 \\
  \text{1.0} & \langle N \rangle > 100 
  \end{cases}
  \]

  \( \delta \) deduced from measured correlations between adjacent pixels
  \( \kappa_2 \) deduced
Expected behavior in asymptotic regimes

- **Ideal gas regime**

\[ \langle \delta N^2 \rangle = \langle N \rangle + \langle N \rangle n \int_{-l_c}^{l_c} dz (g_2(z) - 1) \]

- Non degenerate gas: \( nl_c \ll 1 \)
  \[ \langle \delta N^2 \rangle \simeq \langle N \rangle \]

- Degenerate gas: \( nl_c \gg 1 \)
  \[ \langle \delta N^2 \rangle \simeq \langle N \rangle nl_c = \langle N \rangle^2 l_c / \Delta \]

- **Quasi-bec regime**

Thermodynamic: \( \mu \simeq gn \)

\[ \Rightarrow \langle \delta N^2 \rangle = \Delta T / g \]
Experimental results

\[ T = 15 \text{nK} \approx \frac{\hbar \omega_{\perp}}{10} \]
\[ t \approx 64 \]
\[ \mu \approx 30 \text{nK} \approx 0.2 \, \hbar \omega_{\perp} \]

- Strong bunching effect in the transition region
- Quasi-bEC both in the thermal and quantum regime
Quasi-condensate in the quantum regime

We measure: $\langle \delta N^2 \rangle < \langle N \rangle$

$\langle \delta N^2 \rangle = \langle N \rangle + \langle N \rangle n \int dz (g_2(z) - 1)$ \implies $g^{(2)} < 1$: Quantum regime

- However we still measure thermal excitations

Shot noise term, removed from the $g^{(2)}$ function, IS quantum.

Low momentum phonons ($k \ll T/\mu \xi$): high occupation number

Non trivial quantum fluctuations dominate

For our datas:

$l_T = (T/\mu \xi)^{-1} < 450$ nm

$l_T \ll \Delta, \delta$
From weakly to strongly interacting gases

- **smaller** $t$: smaller bunching $\langle \delta N^2 \rangle_{\text{max}} / \langle N \rangle \propto t^{1/3}$
- $t \ll 1$: Fermi behavior $\rightarrow$ from poissonian to sub-poissonian. No bunching anymore

$t$ small, $\gamma$ large $\Rightarrow g$ large $\Rightarrow$ large $\omega_\perp$
Density fluctuations close to the strongly interacting regime

Moderate compression: \( \omega_\perp / 2\pi = 18.8 \text{ kHz} \)
\[ T = 40 \text{ nK} \approx \hbar \omega_\perp / 20 \]
\[ \mu / T \approx 1.9 \]

No bunching seen anymore, at a level of 20%.
Behavoir close to that of a Fermi gas
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1D-3D Crossover in the quasi-bec regime

\[ \mu \ll \hbar \omega_\perp (na \ll 1) \rightarrow \text{Pure 1D quasi-bec} \]
\[ \mu \gtrsim \hbar \omega_\perp : \text{1D} \rightarrow \text{3D behavior} \]

Transverse breathing associated with a longitudinal phonon has to be taken into account.

Thermodynamic argument: \( \text{Var}(N) = k_B T \left( \frac{\partial N}{\partial \mu} \right)_T \)

Heuristic equation: \( \mu = \hbar \omega_\perp \sqrt{1 + 4na} \)

Efficient thermometry

\[ T = 96 \text{ nK} \simeq 0.5\hbar \omega_\perp \]
Modified Yang-Yang model

If $\mu_{co} \ll \hbar \omega_\perp$, transversally excited states behave as ideal Bose gases.

\[ \mu_{co} \simeq E_g^{1/3} T^{2/3} = \left( \frac{mg^2}{\hbar^2} \right)^{1/3} T^{2/3} \Rightarrow \mu_{co}/(\hbar \omega_\perp) \simeq \left( \frac{T}{\hbar \omega_\perp a/l_\perp} \right)^{2/3} \]

For our parameters: $a/l_\perp \simeq 0.025$

**Modified Yang-Yang model:**

- Transverse ground state: Yang-Yang thermodynamic
- Excited transverse states: ideal 1D Bose gases

Modified Yang-Yang model

\[ T = 150 \text{ nK} \approx 1.0 \hbar \omega_\perp \]

\[ T = 490 \text{ nK} \approx 3.2 \hbar \omega_\perp \]

Very good agreement
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Non gaussian fluctuations

We look at \( \langle \delta N^3 \rangle \)

- **Motivation**
  
  Learn about 3 body correlations

- **A thermodynamic quantity :**

\[
\langle \delta N^3 \rangle = \Delta T^2 \frac{\partial^2 n}{\partial \mu^2}
\]

- **Effect of finite spatial resolution :**

\[
\langle \delta N^3 \rangle = \kappa_3 \langle \delta N^3 \rangle_{\text{true}}, \quad \kappa_3 \text{ depends on resolution.}
\]

\( \kappa_3 \) inferred from neighbour pixels correlation.
Third moment of density fluctuations

$T = 380 \text{ nK}$

$T = 96 \text{ nK}$

Measured 3rd moment compatible with MYY.
Skweness vanishes in quasi-bec regime (as expected)

$\gamma_m = \langle \delta N^3 \rangle / \langle \delta N^2 \rangle^{3/2}$
Three-body correlations

\[ \langle \delta N^3 \rangle \text{ function of } g^{(2)} \text{ and } g^{(3)} \]. \( g^{(3)} \) function contains \( g^{(2)} \).

We measure :

\[
\mathcal{H} = \langle \delta N^3 \rangle + 2\langle N \rangle - 3\langle \delta N^2 \rangle = n^3 \int \int \int h(z_1, z_2, z_3),
\]

\[
h(z_1, z_2, z_3) = g^{(3)}(z_1, z_2, z_3) - g^{(2)}(z_1, z_2) - g^{(2)}(z_1, z_3) - g^{(2)}(z_2, z_3) + 2
\]
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Momentum distribution measurements: motivation

Momentum distribution: TF of first order correlation function

\[ n_k = n \int d\mathbf{u} g^{(1)}(\mathbf{u}) e^{iku} \]

Cannot be derived from the equation of state.

Its measurement opens perspectives (study of dynamics for example)

Problem: global measurement → averaged over many different situations in presence of a smooth longitudinal potential
Focussing techniques

Momentum distribution measurement

• Short kick of a strong harmonic potential
  \[ \delta p = -Az \]

• Free flight until focuss

Final spatial distribution: initial momentum distribution, averaged over the initial position.

Experiment well adapted:
• Longitudinal confinement independent on transverse one
• Purely harmonic potential (up to order \( z^5 \))
Focussing

Images taken at focus \((t_v = 15\text{ms})\) for different initial temperatures (initial RF knife position)
Expected results in limiting cases

Momentum distribution not knowned exactly.

- **Ideal gas**
  - Classical degenerate
  - Thermal quantum classical field theory valid

- **Quasi-condensate**
  - Thermal quantum classical field theory valid

Limited cases:

\[ \frac{1}{\lambda_{dB}} \]
\[ n_{co} \]
\[ n \]
\[ n_{p} \]
\[ p \]
\[ m k_B T \]
\[ \hbar n \]
\[ n_{p} \]
\[ p \]
\[ m k_B T / \hbar n \]
\[ m k_B T / 2 \hbar n \]

Beyond this approach, some knowed results:

- $1/p^4$ tail
- Mean kinetic energy (second moment of $n_p$) via Yang-Yang
1D classical field calculation

\( \psi(z) \): complex field of energy functionnal

\[
E[\{\Psi\}] = \int dz \left[ \frac{\hbar^2}{2m} \left| \frac{d\Psi}{dz} \right|^2 + \frac{g}{2} |\Psi|^4 - \mu |\Psi|^2 \right]
\]

No high energy divergence (as in higher dimension).

Mapped to a Schrödinger equation, evolving in imaginary time

\[
\hat{H} = \frac{\hat{p}^2}{2M} + \frac{\hbar}{k_B T} \left[ \frac{g}{2} (\hat{x}^2 + \hat{y}^2)^2 - \mu (\hat{x}^2 + \hat{y}^2) \right], \quad M = \frac{\hbar^3}{mk_B T}
\]

For \( L \to \infty \), only ground state contribute to \( g^{(1)} \):

\[
g^{(1)}(z) = \langle \phi_0 | (\hat{x} - i\hat{y}) e^{-\hat{H}z/\hbar} (\hat{x} + i\hat{y}) |\phi_0 \rangle
\]

Very fast calculation (as opposed to Monte Carlo sampling).
Comparison with exact calculations

![Graph showing the comparison between different regimes of 1D Bose gases. The graph illustrates the transition from ideal gas to quasi-condensate with classical field theory valid at fixed $T$. The diagram also includes parameters $t = 60, \gamma = 0.027$ for $\mu > 0, n > n_{co}$ and $t = 60, \gamma = 0.056$ for $\mu < 0, n < n_{co}$. The figures on the right show Gaussian or $1/p^4$ tails that are not reproduced by CF.]
Result: degenerate ideal Bose gas regime

- In agreement with in situ density fluctuations
- Good agreement with CF
- Lorentzian behavior: $1/p^2$ tail
Result: quasi-condensate regime

- In agreement with in-situ density fluctuations
- Good agreement with CF
- Lorentzian behavior: $1/p^2$ tail
- Not in agreement with Bogoliubov
Conclusion and prospects

Conclusion

- Precise density fluctuation measurement
  - Good thermometry
  - Investigation of the quasi-condensation transition
  - Strong anti-bunching
  - Higher order correlation functions
  - Dimensional crossover

- Momentum distribution measurement

Prospects

- Investigation of out-of-equilibrium situations: dynamic following a quench, relaxation towards non-thermal states
- Using tomography to gain in spatial resolution and investigate $g^{(2)}(r)$
- Investigating the physics of the Mott transition in 1D using the probes we developed: pinning transition.